Linear Algebra I for Science (NYC)

	Content	Content Type *	Performance Criteria**
	Element No. 1: ***To express concrete problems as linear equations .***		
	To solve systems of linear equations using matrices.		
	Topic: MATRICES		
1.1	Give the definition of a matrix, identify the elements and the size of a matrix.	ESS	10
1.2	Construct a matrix given the general term aij.	ESS	1,3,8,10
1.3	Define equality for matrices and demonstrate its use.	ESS	1,3,7,8,10
1.4	Carry out addition, subtraction, scalar multiplication, transpose and trace of matrices and matrix multiplication.	ESS	1,3,7,8,9,10
1.5	Explain the conditions under which the operations listed in 1.4 can be carried out.	ESS	1,3,6,7,8,9,10
1.6	Identify different types of matrices such as: row, column, square, zero, identity, triangular, diagonal, symmetric and skew-symmetric. Identify the main diagonal of a square matrix.	ESS	2,3,7,8,9,10
1.7	Applications involving the matrix types listed in 1.6.	OPT	1,2,3,6,7,8,9,10
1.8	Use the properties of the matrices listed in 1.6 to prove theorems.	ESS	3,6,7,8,10
1.9	Use Maple or Matlab to perform the operations listed in 1.4.	ESS	3,9,10
	Topic: DETERMINANTS		
1.10	Evaluate a determinant by row and column expansion.	ESS	1,3,7,8,10
1.11	State and use the properties of determinants.	ESS	1,3,6,7,8,9,10
1.12	Determine minors and cofactors.	ESS	7,8,10
1.13	Determine the inverse of a matrix using the cofactor method.	ESS	1,3,6,7,8,9,10
1.14	Solve a system of linear equations using Cramer's Rule.	ESS	1,2,3,4,6,7,8,9,10
1.15	Application of determinants.	OPT	1,2,3,6,7,8,10
1.16	State and/or prove theorems.	ESS	1,2,3,6,7,8,9,10
1.17	Use Maple or Matlab to evaluate a determinant using row and column operations.	ESS	3,9,10
	Topic: SYSTEMS OF LINEAR EQUATIONS		
1.18	Construct the augmented matrix for a system of linear equations.	ESS	2
1.19	Perform elementary row operations on the augmented matrix.	ESS	1,2,3,6,7,8,10
1.20	Define row echelon form and reduced row echelon form.	ESS	1,2,3,10
1.21	Reduce a matrix to row echelon form or reduced row echelon form using elementary row operations.	ESS	1,2,3,6,7,8,10
1.22	Define the rank of a matrix and determine the rank of a given matrix.	ESS	1,2,3,6,7,8,9,10
1.23	Identify homogeneous or non-homogeneous systems of linear equations.	ESS	2
1.24	Determine if a system is consistent or inconsistent.	ESS	1,2,3,4,6,7,8,9,10
1.25	Determine if a system has one solution, no solution or infinitely many solutions and give the general solution in the latter case.	ESS	1,2,3,4,6,7,8,9,10
1.26	Write a system of linear equations in matrix form as AX = B.	ESS	1,2
1.27	Determine the inverse of a matrix by the Gauss-Jordan method.	ESS	1,2,3,7,8,9,10
1.28	Use the inverse to solve a system of linear equations.	ESS	1,2,3,4,7,8,9,10
1.29	Use elementary matrices to determine LU factorization of a matrix A, and use this factorization to solve linear systems with coefficient matrix A.	ESS	1,2,3,4,7,8,9,10
1.30	Express a given problem as a system of linear equations and use these methods to solve the system.	ESS	1,2,3,4,7,8,9,10

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1.31	State and/or prove theorems.	ESS	1,2,3,6,7,8,9,10
1.32	Use Maple or Matlab to solve systems of linear equations.	ESS	3,4,9,10
	Element No. 2: ***To establish connections between geometry and algebra.*** ***To determine the equation of geometric loci (straight lines and planes) and determine their intersections.*** ***To calculate angles, lengths, areas and volumes.*** ***To make two- and three-dimensional drawings of loci.***		
	Topic: PROPERTIES OF ALGEBRAIC AND GEOMETRIC VECTORS		
2.1	Define an algebraic vector as an n-tuple, define equality of vectors.	ESS	1,10
2.2	Carry out the operations of addition, subtraction and scalar multiplication and be able to demonstrate the major properties of these operations.	ESS	1,3,7,8,10
2.3	Determine the magnitude and the direction of a vector and represent vectors graphically.	ESS	1,2,3,5,7,8,9,10
2.4	Use geometric methods to determine equality, sum and difference, scalar multiples of vectors.	ESS	1,2,5,9,10
2.5	Define the scalar product of two vectors and calculate scalar products.	ESS	1,3,7,8,10
2.6	Demonstrate an understanding of the principal properties of the scalar product.	ESS	1,2,7,10
2.7	Use the scalar product to determine the angle between two vectors and the projection of one vector onto another.	ESS	1,2,3,5,7,8,9,10
2.8	Use the scalar product to establish that two vectors are orthogonal.	ESS	1,3,5,7,8,9,10
2.9	Use the scalar product to solve geometric problems.	ESS	1,3,5,7,8,9,10
2.10	State and use Cauchy-Schwarz Inequality and the Triangle Inequalities	ESS	1,2,3,5,6,7,8,9,10
2.11	Define the cross product of two vectors and calculate cross products.	ESS	1,2,3,5,7,8,10
2.12	Demonstrate an understanding of the principal properties of the cross product.	ESS	1,2,7,10
2.13	Use the unit vectors i, j, k.	ESS	1,2,5,10
2.14	Use the cross product to produce a vector orthogonal to two given vectors, to determine the angle between them and to determine if vectors are parallel.	ESS	1,2,3,5,7,8,9,10
2.15	Calculate the area of a parallelogram or triangle determined by two vectors.	ESS	1,3,7,8,9,10
2.16	Define and calculate the scalar triple product.	ESS	1,3,7,8,10
2.17	Demonstrate an understanding of the principal properties of the scalar triple product.	ESS	1,2,7,10
2.18	Determine the volume of a parallelepiped defined by three given vectors.	ESS	1,2,3,5,7,8,9,10
2.19	Test if three given vectors are coplanar.	ESS	1,3,7,8,9,10
2.20	Solve geometric problems using vector methods.	ESS	1,2,3,5,6,7,8,9,10
2.21	Use Maple or Matlab to determine scalar and vector products, the norm of a vector and to calculate angles, lengths, areas and volumes.	ESS	3,9,10

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	Content	Content Type *	Performance Criteria**
	Topic: LINES AND PLANES IN R3		
2.22	Write the symmetric, parametric and vector forms of the equations of a line.	ESS	1,3,10
2.23	Determine points on a given line and test if a given point is on a given line.	ESS	1,3,7,8,9,10
2.24	Determine a direction vector for a line, and write the equations for a line determined by a) two given points, and b) a given point and a given direction vector.	ESS	1,2,3,5,7,8,10
2.25	Determine if two lines intersect, and if they do, determine the point of intersection.	ESS	1,2,3,4,5,7,8,9,10
2.26	Determine if two lines are skew.	ESS	1,2,3,4,5,7,8,9,10
2.27	Calculate the distance from a point to a line and between two skew lines.	ESS	1,2,3,5,7,8,9,10
2.28	Determine the point on a given line closest to a given point.	ESS	1,2,3,5,7,8,9,10
2.29	Write the linear and vector forms of the equation of a plane.	ESS	1,3,10
2.30	Determine a vector normal to a given plane.	ESS	1,2,3,5,7,8,9,10
2.31	Determine points on a given plane and test if a given point lies in a given plane.	ESS	1,3,7,8,9,10
2.32	Write an equation for a plane determined by a) a normal vector and one point in the plane, and b) two vectors and one point in the plane, and c) three points in the plane.	ESS	1,2,3,5,7,8,10
2.33	Determine if two planes are parallel or orthogonal.	ESS	1,2,3,4,5,7,8,9,10
2.34	Determine the angle between two planes.	ESS	1,2,3,5,7,8,9,10
2.35	Determine the intersection of two planes	ESS	1,2,3,4,5,7,8,9,10
2.36	Determine if a line and plane are parallel or orthogonal.	ESS	1,2,3,4,5,7,8,9,10
2.37	Determine the intersection of a line and a plane and the angle between them.	ESS	1,2,3,4,5,7,8,9,10
2.38	Determine the distance from a point to a plane and between two parallel planes.	ESS	1,2,3,4,5,7,8,9,10
2.39	Discuss the possible configurations of three or more planes.	ESS	1,2,5,10
2.40	Determine the point in a given plane closest to a given point.	ESS	1,2,3,5,7,8,9,10
2.41	Sketch lines in Sketch planes in	ESS	5,9,10
2.42	Use Maple or Matlab to determine the equation of a line and of a plane, to make two- and three-dimensional drawings of loci.	OPT	3,9,10
	Topic: VECTOR SPACES		
2.43	Give the definition of a vector space.	ESS	5,10
2.44	Test if a given structure is a vector spaces by verifying the axioms or determining a counter-examples where an axiom fails to hold (examples to include matrices, polynomials, etc.).	ESS	1,2,5,7,9,10
2.45	Define a sub-space and test as above.	ESS	1,2,5,7,9,10
2.46	Define a linear combination of vectors.	ESS	2,5,10
2.47	Determine if a given vector is a linear combination of given set of vectors.	ESS	1,2,3,4,5,6,7,8,9,10

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2.48	Define span of the set of vectors and determine whether the set of vectors spans the vector space.	ESS	1,2,3,6, 7,8,9,10
2.49	Define linear dependence and independence and be able to test a given set of vectors for these properties.	ESS	1,2,3,4,5,6,7,8,9,10
2.50	Interpret geometrically linear combinations, linear independence (dependence) and spanning.	ESS	1,2,3,4,5,6,7,8,9,10
2.51	Define a basis for a vector space or sub-space.	ESS	1,5,10
2.52	Define and find the solution space of a homogeneous linear system, their basis, and dimension.	ESS	1,2,3,4,4,6,7,8,9,10
2.53	Show that a given set of vectors form a basis and determine the components of other vectors relative to this basis	ESS	1,2,3,4,5,6,7,8,9,10
2.54	Determine the dimension of a given vector space or sub-space.	ESS	1,2,3,4,5,6,7,8,9,10
2.55	Define and give examples of orthonormal bases.	ESS	1,2,3,6,7,8,10

	Content	Content Type *	Performance Criteria**
	Topic: Applications of systems of linear equations and matrices (at teacher's discretion)		
3.1	Traffic flow	OPT	1,3,4,6,7,9,9,10
3.2	Electrical circuits.	OPT	1,3,4,6,7,9,9,10
3.3	Balancing chemical reactions	OPT	1,3,4,6,7,9,9,10
3.4	Polynomial interpolation	OPT	1,3,4,6,7,9,9,10
3.5	Leontief Input – Output models	OPT	1,3,4,6,7,9,9,10
3.6	Dynamical systems and Markov chains	OPT	1,3,4,6,7,9,9,10
3.7	Geometric linear programming	OPT	1,3,4,6,7,9,9,10
3.8	Application to computer graphics	OPT	1,3,4,6,7,9,9,10
	Topic: Optional Items		
4.1	Define the null space, column space, and row space of a matrix. Find a basis for, and the dimension for each of these space.	OPT	1,2,3,4,6,7,8,9,10
4.2	Define a linear transformation	OPT	1,10
4.3	Define the kernel and range of a linear transformation	OPT	1,10

Remarks:

1. The numbering of the content in this document is merely for reference purposes. The actual order in which the course material is presented is at the teacher's discretion.
2. At the teacher's discretion, appropriate and closely related definitions, derivations, proofs, and applications using pertinent technology may be added and form part of the evaluation.

	*Content Types
ESS	Essential – Must be covered
OPT	Optional – Suggestions for extra material
PRE	Prerequisite – Suggestions for review material

** Performance Criteria
1. Appropriate use of concepts
2. Adequate representation of situations in vector and matrix forms
3. Correct use of algorithms
4. Correct solution to linear systems
5. Adequate representation of objects in space
6. Justification of steps in problem-resolution procedure
7. Use of algebraic operations in conformity with rules
8. Accuracy of calculations
9. Correct interpretation of results
10. Use of appropriate terminology

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