Specific Elements of the Competency (*in italics*) Standard of Performance: The student must be able to:

1. *Place the development of differential calculus in its historical context.*

1.1 HISTORICAL DEVELOPMENT

1.1.1 describe some of the factors which led to the development of differential calculus.

2. *Recognise and describe the characteristics of algebraic, exponential, logarithmic and trigonometric functions, expressed in symbolic or graphical form.*

2.1 FUNCTIONS

2.1.1 define a function of real numbers and identify if a given graph is the graph of a function.
2.1.2 find domains and ranges of functions, in particular linear, quadratic, polynomial, algebraic, trigonometric, logarithmic and exponential functions.
2.1.3 know the definitions, basic properties and identities for trigonometric functions.
2.1.4 know the definitions, properties and laws of logs and exponentials.
2.1.5 solve logarithmic and exponential equations.
2.1.6 carry out arithmetic operations on polynomials (including long division and factoring).
2.1.7 find the composition of functions, the inverse of a function.
2.1.8 find x and y intercepts and graph linear, quadratic, absolute value and piecewise functions.
2.1.9 graph basic trig, log and exponential functions.
2.1.10 graph a function satisfying certain conditions.

3. *Analyse the behaviour of a function represented symbolically or graphically using an intuitive approach to the concept of limit.*

3.1 LIMITS AND CONTINUITY

3.1.1 calculate two-sided and one-sided limits, limits at infinity and infinite limits from the graph of a function.
3.1.2 use the Limit Theorems to calculate limits as above.
3.1.3 explain why a limit does not exist.
3.1.4 for limits of the form $\frac{0}{0}$, evaluate them by factoring, reducing fractions to a common denominator or rationalizing.
3.1.5 evaluate limits of the form $\frac{\infty}{\infty}$, $\infty - \infty$, $0 \cdot \infty$ by algebraic manipulation.
3.1.6 calculate simple trig limits, in particular making use of $\lim_{x \to 0} \frac{\sin x}{x}$.
3.1.7 define continuity of a function at a point or on an interval.
3.1.8 state the intervals on which a given function is continuous.
3.1.9 discuss the continuity of the functions named specifically above.
3.1.10 identify removable and essential discontinuities and be able to explain the difference using the definition.
3.1.11 graph a function having certain limit and continuity properties.
3.1.12 state the Intermediate and Extreme Value Theorems and use them in examples.

4. *Define the derivative of a function, give its interpretation and apply techniques of differentiation.*

4.1 DIFFERENTIATION

4.1.1 give the definition of the derivative of a function.
4.1.2 interpret the derivative as the slope of the tangent line to the function and as a rate of change; in particular, applications such as marginal cost, revenue, profit.
4.1.3 use the definition to calculate derivatives.
4.1.4 find the equations of tangent lines and normal lines to the graphs of given functions.
4.1.5 apply the rules to differentiate a sum, scalar multiple, product, quotient and composition of functions.
4.1.6 find derivatives of algebraic, trig, log and exponential functions.
4.1.7 use algebraic manipulation to simplify derivatives and determine where $f'(x) = 0$, $f'(x) > 0$, $f'(x) < 0$.
4.1.8 find higher order derivatives.
4.1.9 use implicit differentiation to find first and second derivatives.
4.1.10 state the Mean Value Theorem and use it in examples.
4.1.11 use logarithmic differentiation appropriately

5. *Use differential calculus to analyse the variations of a function and graph it.*

5.1 GRAPH SKETCHING

5.1.1 find the intervals on which a function is increasing or decreasing.
5.1.2 find critical points where $f'(x) = 0$ and where $f'(x)$ does not exist.
5.1.3 find intervals on which a function is concave up or down.
5.1.4 find points of inflection.
5.1.5 find horizontal and vertical asymptotes (with appropriate limits).
5.1.6 use all the information above to sketch the graph of a function.

6. *Solve optimisation and rate of change problems.*

6.1 APPLICATIONS TO SOCIAL SCIENCE

6.1.1 formulate a word problem in mathematical terms and solve it — in particular related rates and maximization/minimization problems involving cost analysis..
6.1.2 solve exponential growth and decay problems — in particular problems about population growth, the spread of epidemics and rumours and the continuous compounding of money..